

mr_math

الرياضيات

/ محمود محمد عبد الرحيم

CONTENTS

Part I	FORMULAS
-------------------	-----------------

	Page
1. Special Constants	1
2. Special Products and Factors	2
3. The Binomial Formula and Binomial Coefficients	3
4. Geometric Formulas	5
5. Trigonometric Functions	11
6. Complex Numbers	21
7. Exponential and Logarithmic Functions	23
8. Hyperbolic Functions	26
9. Solutions of Algebraic Equations	32
10. Formulas from Plane Analytic Geometry	34
11. Special Plane Curves	40
12. Formulas from Solid Analytic Geometry	46
13. Derivatives	53
14. Indefinite Integrals	57
15. Definite Integrals	94
16. The Gamma Function	101
17. The Beta Function	103
18. Basic Differential Equations and Solutions	104
19. Series of Constants	107
20. Taylor Series	110
21. Bernoulli and Euler Numbers	114
22. Formulas from Vector Analysis	116
23. Fourier Series	131
24. Bessel Functions	136
25. Legendre Functions	146
26. Associated Legendre Functions	149
27. Hermite Polynomials	151
28. Laguerre Polynomials	153
29. Associated Laguerre Polynomials	155
30. Chebyshev Polynomials	157

CONTENTS

	Page
31. Hypergeometric Functions	160
32. Laplace Transforms	161
33. Fourier Transforms	174
34. Elliptic Functions	179
35. Miscellaneous Special Functions	183
36. Inequalities	185
37. Partial Fraction Expansions	187
38. Infinite Products	188
39. Probability Distributions	189
40. Special Moments of Inertia	190
41. Conversion Factors	192

Part II	TABLES
--------------------	---------------

Sample problems illustrating use of the tables	194
1. Four Place Common Logarithms	202
2. Four Place Common Antilogarithms	204
3. $\sin x$ (x in degrees and minutes)	206
4. $\cos x$ (x in degrees and minutes)	207
5. $\tan x$ (x in degrees and minutes)	208
6. $\cot x$ (x in degrees and minutes)	209
7. $\sec x$ (x in degrees and minutes)	210
8. $\csc x$ (x in degrees and minutes)	211
9. Natural Trigonometric Functions (in radians)	212
10. $\log \sin x$ (x in degrees and minutes)	216
11. $\log \cos x$ (x in degrees and minutes)	218
12. $\log \tan x$ (x in degrees and minutes)	220
13. Conversion of radians to degrees, minutes and seconds or fractions of a degree	222
14. Conversion of degrees, minutes and seconds to radians	223
15. Natural or Napierian Logarithms $\log_e x$ or $\ln x$	224
16. Exponential functions e^x	226
17. Exponential functions e^{-x}	227
18a. Hyperbolic functions $\sinh x$	228
18b. Hyperbolic functions $\cosh x$	230
18c. Hyperbolic functions $\tanh x$	232

CONTENTS

	Page
19. Factorial n	234
20. Gamma Function	234
21. Binomial Coefficients	236
22. Squares, Cubes, Roots and Reciprocals	238
23. Compound Amount: $(1 + r)^n$	240
24. Present Value of an Amount: $(1 + r)^{-n}$	241
25. Amount of an Annuity: $\frac{(1 + r)^n - 1}{r}$	242
26. Present Value of an Annuity: $\frac{1 - (1 + r)^{-n}}{r}$	243
27. Bessel functions $J_0(x)$	244
28. Bessel functions $J_1(x)$	244
29. Bessel functions $Y_0(x)$	245
30. Bessel functions $Y_1(x)$	245
31. Bessel functions $I_0(x)$	246
32. Bessel functions $I_1(x)$	246
33. Bessel functions $K_0(x)$	247
34. Bessel functions $K_1(x)$	247
35. Bessel functions $\text{Ber}(x)$	248
36. Bessel functions $\text{Bei}(x)$	248
37. Bessel functions $\text{Ker}(x)$	249
38. Bessel functions $\text{Kei}(x)$	249
39. Values for Approximate Zeros of Bessel Functions	250
40. Exponential, Sine and Cosine Integrals	251
41. Legendre Polynomials $P_n(x)$	252
42. Legendre Polynomials $P_n(\cos \theta)$	253
43. Complete Elliptic Integrals of First and Second Kinds	254
44. Incomplete Elliptic Integral of the First Kind	255
45. Incomplete Elliptic Integral of the Second Kind	255
46. Ordinates of the Standard Normal Curve	256
47. Areas under the Standard Normal Curve	257
48. Percentile Values for Student's t Distribution	258
49. Percentile Values for the Chi Square Distribution	259
50. 95th Percentile Values for the F Distribution	260
51. 99th Percentile Values for the F Distribution	261
52. Random Numbers	262
Index of Special Symbols and Notations	263
Index	265

Part I

FORMULAS

THE GREEK ALPHABET

Greek name	Greek letter	
	Lower case	Capital
Alpha	α	A
Beta	β	B
Gamma	γ	Γ
Delta	δ	Δ
Epsilon	ϵ	E
Zeta	ζ	Z
Eta	η	H
Theta	θ	Θ
Iota	ι	I
Kappa	κ	K
Lambda	λ	Λ
Mu	μ	M

Greek name	Greek letter	
	Lower case	Capital
Nu	ν	N
Xi	ξ	Ξ
Omicron	\omicron	O
Pi	π	Π
Rho	ρ	P
Sigma	σ	Σ
Tau	τ	T
Upsilon	υ	Υ
Phi	ϕ	Φ
Chi	χ	X
Psi	ψ	Ψ
Omega	ω	Ω

1

SPECIAL CONSTANTS

- 1.1 $\pi = 3.14159\ 26535\ 89793\ 23846\ 2643\ 326795\ 288417\ 699437510\ 5820974944\ 592307864086910841$
- 1.2 $e = 2.71828\ 18284\ 59045\ 23536\ 0287\dots = \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n$
= natural base of logarithms
- 1.3 $\sqrt{2} = 1.41421\ 35623\ 73095\ 04882\dots$
- 1.4 $\sqrt{3} = 1.73205\ 08075\ 68877\ 2935\dots$
- 1.5 $\sqrt{5} = 2.23606\ 79774\ 99789\ 6964\dots$
- 1.6 $\sqrt[3]{2} = 1.25992\ 1050\dots$
- 1.7 $\sqrt[3]{3} = 1.44224\ 9570\dots$
- 1.8 $\sqrt[5]{2} = 1.14869\ 8355\dots$
- 1.9 $\sqrt[5]{3} = 1.24573\ 0940\dots$
- 1.10 $e^\pi = 23.14069\ 26327\ 79269\ 006\dots$
- 1.11 $\pi^e = 22.45915\ 77183\ 61045\ 47342\ 715\dots$
- 1.12 $e^e = 15.15426\ 22414\ 79264\ 190\dots$
- 1.13 $\log_{10} 2 = 0.30102\ 99956\ 63981\ 19521\ 37389\dots$
- 1.14 $\log_{10} 3 = 0.47712\ 12547\ 19662\ 43729\ 50279\dots$
- 1.15 $\log_{10} e = 0.43429\ 44819\ 03251\ 82765\dots$
- 1.16 $\log_{10} \pi = 0.49714\ 98726\ 94133\ 85435\ 12683\dots$
- 1.17 $\log_e 10 = \ln 10 = 2.30258\ 50929\ 94045\ 68401\ 7991\dots$
- 1.18 $\log_e 2 = \ln 2 = 0.69314\ 71805\ 59945\ 30941\ 7232\dots$
- 1.19 $\log_e 3 = \ln 3 = 1.09861\ 22886\ 68109\ 69139\ 5245\dots$
- 1.20 $\gamma = 0.57721\ 56649\ 01532\ 86060\ 6512\dots = \text{Euler's constant}$
 $= \lim_{n \rightarrow \infty} \left(1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n} - \ln n\right)$
- 1.21 $e^\gamma = 1.78107\ 24179\ 90197\ 9852\dots$ [see 1.20]
- 1.22 $\sqrt{e} = 1.64872\ 12707\ 00128\ 1468\dots$
- 1.23 $\sqrt{\pi} = \Gamma(\frac{1}{2}) = 1.77245\ 38509\ 05516\ 02729\ 8167\dots$
where Γ is the *gamma function* [see pages 101-102].
- 1.24 $\Gamma(\frac{1}{3}) = 2.67893\ 85347\ 07748\dots$
- 1.25 $\Gamma(\frac{1}{4}) = 3.62560\ 99082\ 21908\dots$
- 1.26 $1 \text{ radian} = 180^\circ/\pi = 57.29577\ 95130\ 8232\dots^\circ$
- 1.27 $1^\circ = \pi/180 \text{ radians} = 0.01745\ 32925\ 19943\ 29576\ 92\dots \text{radians}$

2

SPECIAL PRODUCTS and FACTORS

- 2.1** $(x + y)^2 = x^2 + 2xy + y^2$
2.2 $(x - y)^2 = x^2 - 2xy + y^2$
2.3 $(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$
2.4 $(x - y)^3 = x^3 - 3x^2y + 3xy^2 - y^3$
2.5 $(x + y)^4 = x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$
2.6 $(x - y)^4 = x^4 - 4x^3y + 6x^2y^2 - 4xy^3 + y^4$
2.7 $(x + y)^5 = x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + y^5$
2.8 $(x - y)^5 = x^5 - 5x^4y + 10x^3y^2 - 10x^2y^3 + 5xy^4 - y^5$
2.9 $(x + y)^6 = x^6 + 6x^5y + 15x^4y^2 + 20x^3y^3 + 15x^2y^4 + 6xy^5 + y^6$
2.10 $(x - y)^6 = x^6 - 6x^5y + 15x^4y^2 - 20x^3y^3 + 15x^2y^4 - 6xy^5 + y^6$

The results 2.1 to 2.10 above are special cases of the *binomial formula* [see page 3].

- 2.11** $x^2 - y^2 = (x - y)(x + y)$
2.12 $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$
2.13 $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$
2.14 $x^4 - y^4 = (x - y)(x + y)(x^2 + y^2)$
2.15 $x^5 - y^5 = (x - y)(x^4 + x^3y + x^2y^2 + xy^3 + y^4)$
2.16 $x^5 + y^5 = (x + y)(x^4 - x^3y + x^2y^2 - xy^3 + y^4)$
2.17 $x^6 - y^6 = (x - y)(x + y)(x^2 + xy + y^2)(x^2 - xy + y^2)$
2.18 $x^4 + x^2y^2 + y^4 = (x^2 + xy + y^2)(x^2 - xy + y^2)$
2.19 $x^4 + 4y^4 = (x^2 + 2xy + 2y^2)(x^2 - 2xy + 2y^2)$

Some generalizations of the above are given by the following results where n is a positive integer.

- 2.20** $x^{2n+1} - y^{2n+1} = (x - y)(x^{2n} + x^{2n-1}y + x^{2n-2}y^2 + \cdots + y^{2n})$

$$= (x - y) \left(x^2 - 2xy \cos \frac{2\pi}{2n+1} + y^2 \right) \left(x^2 - 2xy \cos \frac{4\pi}{2n+1} + y^2 \right) \cdots \left(x^2 - 2xy \cos \frac{2n\pi}{2n+1} + y^2 \right)$$

2.21 $x^{2n+1} + y^{2n+1} = (x + y)(x^{2n} - x^{2n-1}y + x^{2n-2}y^2 - \cdots + y^{2n})$

$$= (x + y) \left(x^2 + 2xy \cos \frac{2\pi}{2n+1} + y^2 \right) \left(x^2 + 2xy \cos \frac{4\pi}{2n+1} + y^2 \right) \cdots \left(x^2 + 2xy \cos \frac{2n\pi}{2n+1} + y^2 \right)$$

2.22 $x^{2n} - y^{2n} = (x - y)(x + y)(x^{n-1} + x^{n-2}y + x^{n-3}y^2 + \cdots)(x^{n-1} - x^{n-2}y + x^{n-3}y^2 - \cdots)$

$$= (x - y)(x + y) \left(x^2 - 2xy \cos \frac{\pi}{n} + y^2 \right) \left(x^2 - 2xy \cos \frac{2\pi}{n} + y^2 \right) \cdots \left(x^2 - 2xy \cos \frac{(n-1)\pi}{n} + y^2 \right)$$

2.23 $x^{2n} + y^{2n} = \left(x^2 + 2xy \cos \frac{\pi}{2n} + y^2 \right) \left(x^2 + 2xy \cos \frac{3\pi}{2n} + y^2 \right) \cdots \left(x^2 + 2xy \cos \frac{(2n-1)\pi}{2n} + y^2 \right)$

3

The BINOMIAL FORMULA and BINOMIAL COEFFICIENTS

FACTORIAL n

If $n = 1, 2, 3, \dots$ *factorial n* or *n factorial* is defined as

$$3.1 \quad n! = 1 \cdot 2 \cdot 3 \cdot \dots \cdot n$$

We also define *zero factorial* as

$$3.2 \quad 0! = 1$$

BINOMIAL FORMULA FOR POSITIVE INTEGRAL n

If $n = 1, 2, 3, \dots$ then

$$3.3 \quad (x + y)^n = x^n + nx^{n-1}y + \frac{n(n-1)}{2!}x^{n-2}y^2 + \frac{n(n-1)(n-2)}{3!}x^{n-3}y^3 + \dots + y^n$$

This is called the *binomial formula*. It can be extended to other values of n and then is an infinite series [see *Binomial Series*, page 110].

BINOMIAL COEFFICIENTS

The result 3.3 can also be written

$$3.4 \quad (x + y)^n = x^n + \binom{n}{1}x^{n-1}y + \binom{n}{2}x^{n-2}y^2 + \binom{n}{3}x^{n-3}y^3 + \dots + \binom{n}{n}y^n$$

where the coefficients, called *binomial coefficients*, are given by

$$3.5 \quad \binom{n}{k} = \frac{n(n-1)(n-2)\dots(n-k+1)}{k!} = \frac{n!}{k!(n-k)!} = \binom{n}{n-k}$$

PROPERTIES OF BINOMIAL COEFFICIENTS

$$3.6 \quad \binom{n}{k} + \binom{n}{k+1} = \binom{n+1}{k+1}$$

This leads to *Pascal's triangle* [see page 236].

$$3.7 \quad \binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \cdots + \binom{n}{n} = 2^n$$

$$3.8 \quad \binom{n}{0} - \binom{n}{1} + \binom{n}{2} - \cdots + (-1)^n \binom{n}{n} = 0$$

$$3.9 \quad \binom{n}{n} + \binom{n+1}{n} + \binom{n+2}{n} + \cdots + \binom{n+m}{n} = \binom{n+m+1}{n+1}$$

$$3.10 \quad \binom{n}{0} + \binom{n}{2} + \binom{n}{4} + \cdots = 2^{n-1}$$

$$3.11 \quad \binom{n}{1} + \binom{n}{3} + \binom{n}{5} + \cdots = 2^{n-1}$$

$$3.12 \quad \binom{n}{0}^2 + \binom{n}{1}^2 + \binom{n}{2}^2 + \cdots + \binom{n}{n}^2 = \binom{2n}{n}$$

$$3.13 \quad \binom{m}{0} \binom{n}{p} + \binom{m}{1} \binom{n}{p-1} + \cdots + \binom{m}{p} \binom{n}{0} = \binom{m+n}{p}$$

$$3.14 \quad (1) \binom{n}{1} + (2) \binom{n}{2} + (3) \binom{n}{3} + \cdots + (n) \binom{n}{n} = n 2^{n-1}$$

$$3.15 \quad (1) \binom{n}{1} - (2) \binom{n}{2} + (3) \binom{n}{3} - \cdots + (-1)^{n+1} (n) \binom{n}{n} = 0$$

MULTINOMIAL FORMULA

$$3.16 \quad (x_1 + x_2 + \cdots + x_p)^n = \sum \frac{n!}{n_1! n_2! \cdots n_p!} x_1^{n_1} x_2^{n_2} \cdots x_p^{n_p}$$

where the sum, denoted by Σ , is taken over all nonnegative integers n_1, n_2, \dots, n_p for which $n_1 + n_2 + \cdots + n_p = n$.

4

GEOMETRIC FORMULAS

RECTANGLE OF LENGTH b AND WIDTH a

4.1 Area = ab

4.2 Perimeter = $2a + 2b$

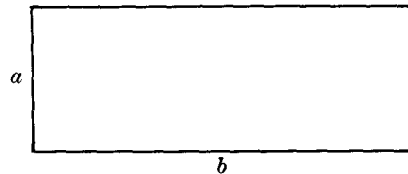


Fig. 4-1

PARALLELOGRAM OF ALTITUDE h AND BASE b

4.3 Area = $bh = ab \sin \theta$

4.4 Perimeter = $2a + 2b$

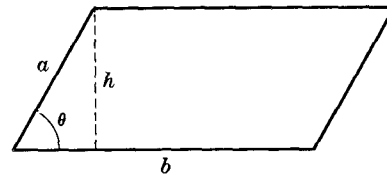


Fig. 4-2

TRIANGLE OF ALTITUDE h AND BASE b

4.5 Area = $\frac{1}{2}bh = \frac{1}{2}ab \sin \theta$
 $= \sqrt{s(s-a)(s-b)(s-c)}$
 where $s = \frac{1}{2}(a+b+c)$ = semiperimeter

4.6 Perimeter = $a + b + c$

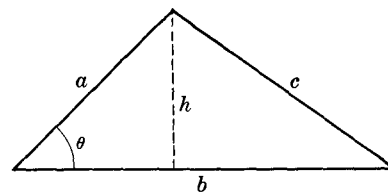


Fig. 4-3

TRAPEZOID OF ALTITUDE h AND PARALLEL SIDES a AND b

4.7 Area = $\frac{1}{2}h(a+b)$

4.8 Perimeter = $a + b + h \left(\frac{1}{\sin \theta} + \frac{1}{\sin \phi} \right)$
 $= a + b + h(\csc \theta + \csc \phi)$

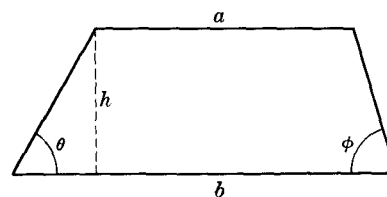


Fig. 4-4

REGULAR POLYGON OF n SIDES EACH OF LENGTH b

$$4.9 \quad \text{Area} = \frac{1}{4}nb^2 \cot \frac{\pi}{n} = \frac{1}{4}nb^2 \frac{\cos(\pi/n)}{\sin(\pi/n)}$$

$$4.10 \quad \text{Perimeter} = nb$$

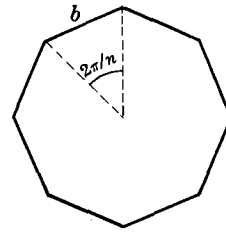


Fig. 4-5

CIRCLE OF RADIUS r

$$4.11 \quad \text{Area} = \pi r^2$$

$$4.12 \quad \text{Perimeter} = 2\pi r$$

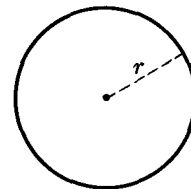


Fig. 4-6

SECTOR OF CIRCLE OF RADIUS r

$$4.13 \quad \text{Area} = \frac{1}{2}r^2\theta \quad [\theta \text{ in radians}]$$

$$4.14 \quad \text{Arc length } s = r\theta$$

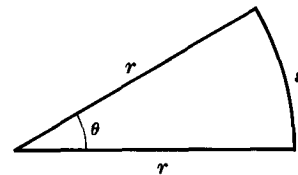


Fig. 4-7

RADIUS OF CIRCLE INSCRIBED IN A TRIANGLE OF SIDES a, b, c

$$4.15 \quad r = \frac{\sqrt{s(s-a)(s-b)(s-c)}}{s}$$

where $s = \frac{1}{2}(a + b + c) = \text{semiperimeter}$

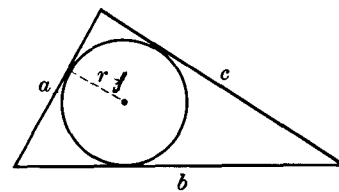


Fig. 4-8

RADIUS OF CIRCLE CIRCUMSCRIBING A TRIANGLE OF SIDES a, b, c

$$4.16 \quad R = \frac{abc}{4\sqrt{s(s-a)(s-b)(s-c)}}$$

where $s = \frac{1}{2}(a + b + c) = \text{semiperimeter}$

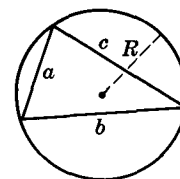


Fig. 4-9

REGULAR POLYGON OF n SIDES INSCRIBED IN CIRCLE OF RADIUS r

$$4.17 \quad \text{Area} = \frac{1}{2}nr^2 \sin \frac{2\pi}{n} = \frac{1}{2}nr^2 \sin \frac{360^\circ}{n}$$

$$4.18 \quad \text{Perimeter} = 2nr \sin \frac{\pi}{n} = 2nr \sin \frac{180^\circ}{n}$$

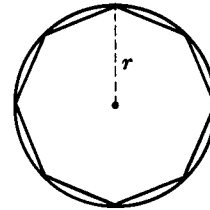


Fig. 4-10

REGULAR POLYGON OF n SIDES CIRCUMSCRIBING A CIRCLE OF RADIUS r

$$4.19 \quad \text{Area} = nr^2 \tan \frac{\pi}{n} = nr^2 \tan \frac{180^\circ}{n}$$

$$4.20 \quad \text{Perimeter} = 2nr \tan \frac{\pi}{n} = 2nr \tan \frac{180^\circ}{n}$$

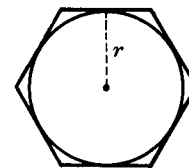


Fig. 4-11

SEGMENT OF CIRCLE OF RADIUS r

$$4.21 \quad \text{Area of shaded part} = \frac{1}{2}r^2 (\theta - \sin \theta)$$

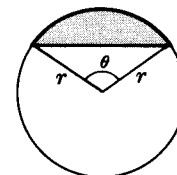


Fig. 4-12

ELLIPSE OF SEMI-MAJOR AXIS a AND SEMI-MINOR AXIS b

$$4.22 \quad \text{Area} = \pi ab$$

$$4.23 \quad \text{Perimeter} = 4a \int_0^{\pi/2} \sqrt{1 - k^2 \sin^2 \theta} d\theta$$

$$= 2\pi \sqrt{\frac{1}{2}(a^2 + b^2)} \quad [\text{approximately}]$$

where $k = \sqrt{a^2 - b^2}/a$. See page 254 for numerical tables.

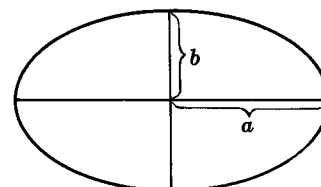


Fig. 4-13

SEGMENT OF A PARABOLA

$$4.24 \quad \text{Area} = \frac{2}{3}ab$$

$$4.25 \quad \text{Arc length } ABC = \frac{1}{2} \sqrt{b^2 + 16a^2} + \frac{b^2}{8a} \ln \left(\frac{4a + \sqrt{b^2 + 16a^2}}{b} \right)$$

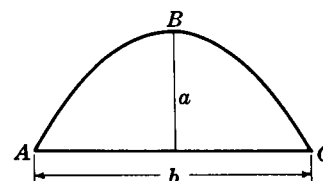


Fig. 4-14

RECTANGULAR PARALLELEPIPED OF LENGTH a , HEIGHT l , WIDTH c

4.26 Volume = abc

4.27 Surface area = $2(ab + ac + bc)$

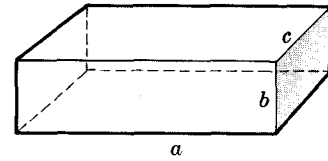


Fig. 4-15

PARALLELEPIPED OF CROSS-SECTIONAL AREA A AND HEIGHT h

4.28 Volume = $Ah = abc \sin \theta$

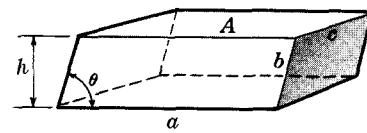


Fig. 4-16

SPHERE OF RADIUS r

4.29 Volume = $\frac{4}{3}\pi r^3$

4.30 Surface area = $4\pi r^2$

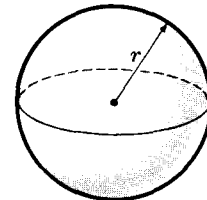


Fig. 4-17

RIGHT CIRCULAR CYLINDER OF RADIUS r AND HEIGHT h

4.31 Volume = $\pi r^2 h$

4.32 Lateral surface area = $2\pi r h$

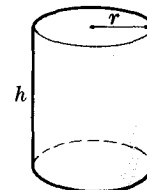


Fig. 4-18

CIRCULAR CYLINDER OF RADIUS r AND SLANT HEIGHT l

4.33 Volume = $\pi r^2 h = \pi r^2 l \sin \theta$

4.34 Lateral surface area = $2\pi r l = \frac{2\pi r h}{\sin \theta} = 2\pi r h \csc \theta$

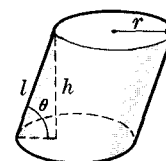


Fig. 4-19

CYLINDER OF CROSS-SECTIONAL AREA A AND SLANT HEIGHT l

4.35 Volume = $Ah = Al \sin \theta$

4.36 Lateral surface area = $pl = \frac{ph}{\sin \theta} = ph \csc \theta$

Note that formulas 4.31 to 4.34 are special cases.

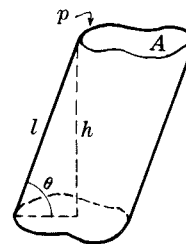


Fig. 4-20

RIGHT CIRCULAR CONE OF RADIUS r AND HEIGHT h

4.37 Volume = $\frac{1}{3}\pi r^2 h$

4.38 Lateral surface area = $\pi r \sqrt{r^2 + h^2} = \pi r l$

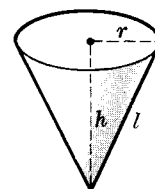


Fig. 4-21

PYRAMID OF BASE AREA A AND HEIGHT h

4.39 Volume = $\frac{1}{3}Ah$

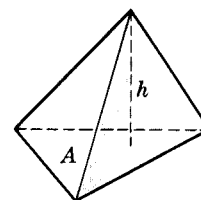


Fig. 4-22

SPHERICAL CAP OF RADIUS r AND HEIGHT h

4.40 Volume (shaded in figure) = $\frac{1}{3}\pi h^2(3r - h)$

4.41 Surface area = $2\pi r h$

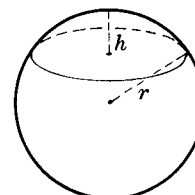


Fig. 4-23

FRUSTRUM OF RIGHT CIRCULAR CONE OF RADII a, b AND HEIGHT h

4.42 Volume = $\frac{1}{3}\pi h(a^2 + ab + b^2)$

4.43 Lateral surface area = $\pi(a + b)\sqrt{h^2 + (b - a)^2}$
= $\pi(a + b)l$

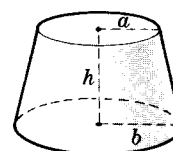


Fig. 4-24

SPHERICAL TRIANGLE OF ANGLES A, B, C ON SPHERE OF RADIUS r

4.44 Area of triangle $ABC = (A + B + C - \pi)r^2$

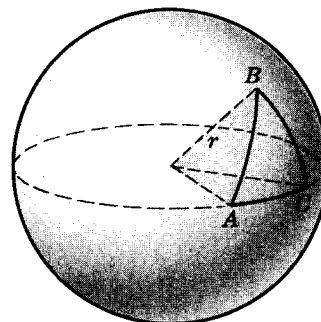


Fig. 4-25

TORUS OF INNER RADIUS a AND OUTER RADIUS b

4.45 Volume = $\frac{1}{4}\pi^2(a+b)(b-a)^2$

4.46 Surface area = $\pi^2(b^2 - a^2)$

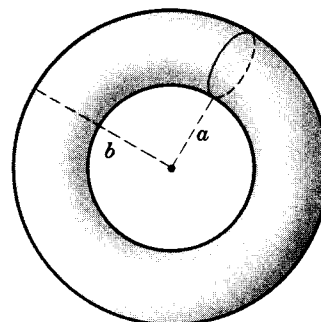


Fig. 4-26

ELLIPSOID OF SEMI-AXES a, b, c

4.47 Volume = $\frac{4}{3}\pi abc$

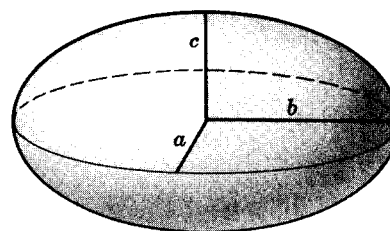


Fig. 4-27

PARABOLOID OF REVOLUTION

4.48 Volume = $\frac{1}{2}\pi b^2 a$

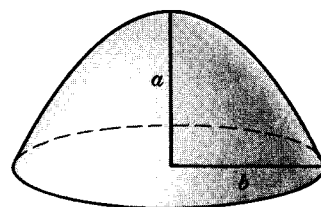


Fig. 4-28